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A novel method for non-parametric identification of nonlinear restoring forces in nonlinear vibrations from noisy response data: A conservative system †

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Abstract

A novel procedure is proposed to identify the functional form of nonlinear restoring forces in the nonlinear oscillatory motion of a conservative system. Although the problem of identification has a unique solution, formulation results in a Volterra-type of integral equation of the "first" kind: the solution lacks stability because the integral equation is the "first" kind. Thus, the new problem at hand is ill-posed. Inevitable small errors during the identification procedure can make the prediction of nonlinear restoring forces useless. We overcome the difficulty by using a stabilization technique of Landweber's regularization in this study. The capability of the proposed procedure is investigated through numerical examples.

Keywords: Identification; Nonlinear restoring forces; Volterra nonlinear integral equation; numerical instability; Regularization method

1. Introduction

Considerable attention has been focused recently on the model identification of dynamic systems in various branches of science and engineering, especially in the vibration engineering field. It is crucial to obtain the correct modeling of a system's nonlinear restoring forces to achieve an accurate prediction of its motion response to various loading environments, since different types of nonlinear restoring forces produce different physical phenomena of a system.

A number of identification methods on this topic are now available. For example, based on neural networks, a procedure to identify the restoring forces of a number of typical nonlinear structural systems was proposed by Masri et al. [1]. The proposed procedure was extended to multi-degree-of-freedom nonlinear vibration systems, and was developed for a wider applicable range by Chassiakos and Masri [2] and Liang et al. [3]. A fuzzy adaptive neural network was applied to identify nonlinear characteristics in nonlinear vibration systems (Liang et al. [4]). Based on the application of the Hilbert transforms, Spina et al. [5] presented a new technique to identify nonlinearity. Several other publications focus on challenging issues in modeling and identifying nonlinear systems [6-10]. However, in most of the methods presented, there are a number of unavoidable limitations: *a priori* information about the functional form under investigation should be required and the properties of the identified system are constrained, and so on. Therefore, a novel method needs to be introduced to improve the estimation of dynamic systems.

In this study, we establish a method that can identify a nonlinear restoring function of a nonlinear system by measuring the dynamic responses. The method we propose is new to this field. Remarkable improvements are made when compared with the

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previous studies.

First, this paper presents a new "nonparametric" identification method. So far, the usual "parametric" identifications are involved in determining individual parameters of pre-assumed models. Thus, the shape of the restoring function should be assumed in advance. However, the method proposed here does not require any *a priori* information on the model for the nonlinear restoring function.

Second, we formulate a new inverse problem for the nonparametric identification, which is mathematically, a nonlinear Volterra integral equation of the first kind: the differential equation for the motion equation is transformed into a nonlinear integral equation. The inverse problem has a unique solution, but it is ill-posed in the sense of stability. This lack of stability in the solution leads to erroneous results when using any conventional numerical approach [11, 12].

Third, we introduce a stabilization technique, known as the regularization method, to suppress the numerical instability in the solution; Landweber's regularization method is applied to stabilize the numerical solution [13-20]. The L-curve criteria [21], combined with the Landweber's regularization method, is also introduced to determine a proper choice of regularization parameters (or number of iterations) for the identification.

The practicality of the proposed procedure is examined through numerical experiments. It is found that the numerical experiment demonstrates the applicability of the proposed method to identify a functional form of nonlinear forces in nonlinear oscillations.

2. Equation of motion

A nonlinear oscillator of a conservative system is considered, in which the governing equation for the oscillation is described by the ordinary differential equation:

$$m\ddot{y} + ky = h(y) \tag{1}$$

Here, m denotes the mass of a particle of an oscillator, k is a spring coefficient, and $h(y)$ is a nonlinear restoring force. Since an integral equation sometimes has advantages over a differential equation, we will transform Eq. (1) into an integral equation. Let us take the initial conditions as in Eq. (2)

$$y(0) = \alpha\ , \quad \dot{y}(0) = \beta \tag{2}$$

The solution of the oscillator is then satisfied by the nonlinear Volterra integral equation [22, 25]:

$$y(t) = \alpha y_1(t) + \beta y_2(t) + \int_0^t \frac{y_1(\tau)y_2(t) - y_1(t)y_2(\tau)}{mW(\tau)} h(y)d\tau \tag{3}$$

where $y_1(t)$ and $y_2(t)$ are chosen so that

$$\begin{aligned} m\ddot{y}_1 + ky_1 &= 0 \quad y_1(0) = 1, \dot{y}_1(0) = 0 \\ m\ddot{y}_2 + ky_2 &= 0 \quad y_2(0) = 0, \dot{y}_2(0) = 1 \end{aligned} \tag{4}$$

and the Wronskian is defined as $W = y_1\dot{y}_2 - \dot{y}_1 y_2$.

3. Inverse problem and unique solution

If a displacement $y(t)$ is measured during the oscillation with the initial conditions in Eq. (2), then an integral Eq. (5) for u can be constructed from Eq. (3) because the left-hand side of Eq. (5) is assumed to be known,

$$y(t) - \alpha y_1(t) - \beta y_2(t) = \int_0^t K(t,\tau)u(\tau)d\tau \tag{5}$$

where u and K are defined, respectively, as

$$u(t) = h[y(t)] \tag{6}$$

$$K(t,\tau) = \frac{y_1(\tau)y_2(t) - y_1(t)y_2(\tau)}{mW(\tau)} \tag{7}$$

It may be important to examine the uniqueness of the solution u to the integral Eq. (5). The integral Eq. (5) is linear in u, thus, it is sufficient to prove that the removal of the left-hand side in Eq. (5), that is, $y(t) - \alpha y_1(t) - \beta y_2(t) = 0$ means that $u = 0$ in Eq. (5).

First, assume that $y(t) - \alpha y_1(t) - \beta y_2(t) = 0$, then we have $y(t) = \alpha y_1(t) + \beta y_2(t)$. Since Eq. (1) is equivalent to Eq. (5), y is the solution to not only Eq. (1) but also Eq. (5). Thus, it is possible to substitute $y(t) = \alpha y_1(t) + \beta y_2(t)$ into Eq. (1) instead of Eq. (5). This substitution leads to

$$0 = h(y) \tag{8}$$

because $y(t) = \alpha y_1(t) + \beta y_2(t)$ is a harmonic solution such that $m\ddot{y} + ky = 0$ due to Eq. (4). The equality in Eq. (8) should hold for any y : the initial conditions for y are given as Eq. (2), in which the values α and β are arbitrary. Eq. (6) indicates that u

(t) is zero because of Eq. (8). Therefore, we have u = 0. This completes the proof. It can be concluded that the integral Eq. (5) has a unique solution.

4. Instability in the solution

While it is found that the inverse problem for nonlinear restoring forces has a unique solution, there remains the question of stability in the sense that the solution to the inverse problem depends continuously on the measured response data.

Superficially, Eqs. (3) and (5) appear to be the same type of integral equations, yet their characteristics for the solution's stability differ. As a forward problem, Eq. (3) is classified as a nonlinear Volterra integral equation of the second kind for y. Since second-kind integral equations are known to be well-posed in the sense of stability [11], conventional numerical methods such as a direct numerical discretization can be considered as the best strategy to solve Eq. (3). In contrast to the forward problem of Eq. (3), an inverse formulation of Eq. (5) has the form of a Volterra-type of integral equation of the first kind for u. The theory of first-kind integral equations illustrates that the first-kind integral equation with the regular kernel such as K in Eq. (7) is ill-posed in the sense of stability [13]. Specifically, we need to deal with the first-kind integral equation for u when recovering nonlinear restoring forces in the inverse problem. This results in numerical instability, which will affect the performance of the present inverse problem.

5. Identifying nonlinear restoring

The identification is an ill-posed inverse problem in the sense that its solution lacks stability properties: a small amount of noisy data can be considerably amplified and may lead to unreliable solutions. To overcome this problem, we suggest using so-called regularization methods, such as Landweber's regularization, to counter the instability of the problem. In fact, a direct discretization of the right-hand side in Eq. (5) creates a matrix in which the condition number is extremely large, so that the numerical inverse of the matrix does not work because the determinant of the matrix is nearly zero [16-20].

For the solution of the integral Eq. (5), we first need to determine the left-hand side in Eq. (5), which will be denoted by η:

$$\eta(t) = y(t) - \alpha y_1(t) - \beta y_2(t) \tag{9}$$

If the motion response y is measured, then the η in Eq. (9) can be calculated using the initial data α,β in Eq. (2) and y_1,y_2, which satisfies Eq. (4). The solution u (t) to the first-kind integral Eq. (5) can be realized by the following iteration, known as Landweber's regularization [24]:

$$u_m = (I - \lambda L^* L)u_{m-1} + \lambda L^* \eta\,,\ \ m = 1, 2... \tag{10}$$

for a real positive constant λ such that $0 < \lambda < 1/\|L\|_2^2$ where $\|\cdot\|_2$ refers to the L_2 norm [11]. In Eq. (10). The symbol L denotes an operator, defined as follows, for an arbitrary function b (t),

$$Lb = \int_0^t K(t,\tau)b(\tau)d\tau \tag{11}$$

L^* represents the adjoint operator of L, and I represents the identity operator [11, 13]: the kernel K in Eq. (11) is the same as in Eq. (7).

The detailed Landweber's regularization theory is explained [13] as follows. Iterative methods to solve equations are popular because they require only relatively simple operations to be performed repeatedly. Many iterative methods can be, and are, applied to ill-posed problems. In this study, we treat only the simplest method, which is Landweber's regularization (10) [13]. It is derived from a second-kind integral equation based on Banach's fixed-point theorem, which guarantees the solution's stability. For convenience, we start with the initial guess of zero function without loss of generality [16-20] because Landweber's regularization converges to the solution for an arbitrary initial guess u_0 [13]:

$$u_0 = 0 \tag{12}$$

Finally, the nonlinear force h (y) in Eq. (1) can be determined from Eq. (6): u (t) in Eq. (6) is a known quantity from the calculation using Eq. (10), while y (t) in Eq. (6) is also a known quantity from the measurement, which enables the functional form h (y) to be determined (algebraically).

6. Numerical experiment: Duffing's equation

In this section, we will demonstrate the applicability of the proposed scheme to identify a functional form of nonlinear restoring force through a numerical experiment.

6.1 Duffing's equation

As a conservative nonlinear oscillation system, Duffing's equation [23] is chosen as a model equation in this paper:

$$m\ddot{y} + ky = \gamma y^3 \tag{13}$$

Comparing Eq. (1) and Eq. (13), h (y) in Eq. (1) takes the form of Eq. (14)

$$h(y) = \gamma y^3 \tag{14}$$

For convenience, the initial conditions for the present numerical experiment are given as in Eq. (15),

$$y(0) = 0, \dot{y}(0) = 1 \tag{15}$$

and the coefficients m and k of Duffing's equation are normalized as a unit.

Numerical integration methods of the initial value problem in the ordinary differential Eq. (13) such as the Runge-Kutta type of integration schemes make it possible to obtain the numerical solution of Eq. (13) using the initial conditions of Eq. (15). The solutions and their phase paths in the phase diagram are depicted for γ = -1 and -3 as shown in Figs. 1-4. The effects of increasing nonlinearity and closed paths in the conservative system are observed.

Now, we consider the inverse problem for detecting nonlinear restoring forces. If we measure the motion response data such as the results shown in Figs. 1 and 2, we can inquire whether it is possible to recover the functional form of the nonlinear restoring force in Eq. (14) from the measured data. To respond to this inquiry, we follow the inverse process as described in Section 6.

6.2 Noisy data

To apply the Landweber's regularization in Eq. (10), measured data of motion response should be given. However, in practice, noise always causes some deterioration to measured data. The left-hand side in Eq. (5) is never exactly known but only up to an error of, say, noise level $\delta > 0$. In this paper, it is assumed that we know $\delta > 0$ and noisy data η^δ with

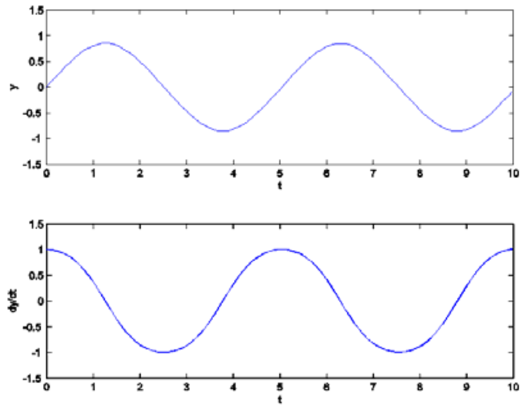


Fig. 1. Motion response of Duffing's equation in Eq. (13) with initial conditions in Eq. (15) for $\gamma = -1$.

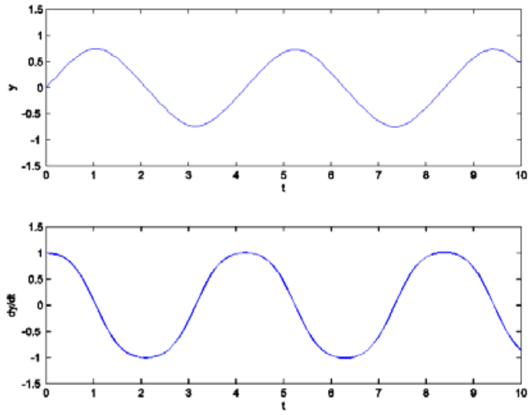


Fig. 2. Motion response of Duffing's equation in Eq. (13) with initial conditions in Eq. (15) for $\gamma = -3$.

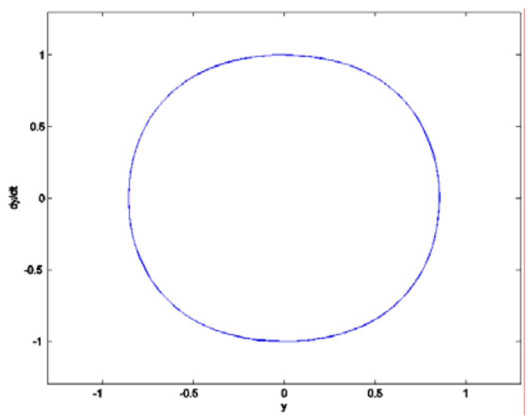


Fig. 3. Phase plane for $\gamma = -1$.

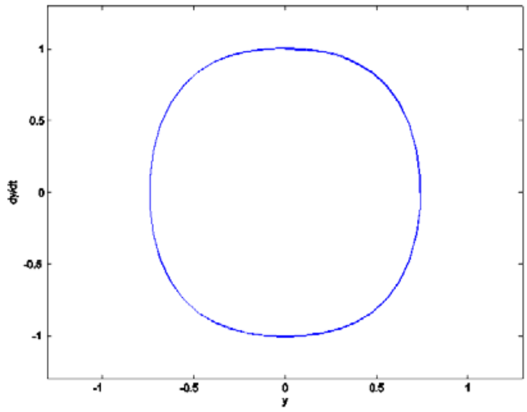


Fig. 4. Phase plane for $\gamma = -3$.

$$\left\| \eta - \eta^{\delta} \right\|_2 \leq \delta \tag{16}$$

in which $\|\cdot\|_2$ denotes L_2 norm [11].

It is now our aim to solve the perturbed equation:

$$\eta^{\delta} = \int_0^t K(t,\tau)u(\tau)d\tau \tag{17}$$

In the present numerical experiments, noisy data is randomly generated for the noise level (or error intensity) of $\delta = 0.0096$.

6.3 L-curve criterion: the optimal choice for the number of iterations

The number of iterations plays an important role in the iterative method of Landweber's regularization. When the number of iterations increases, the iterated solutions approach the true solution in the initial stage of the iteration, and subsequently, potentially deviate far away. In the inverse process, the accuracy of the Landweber's iteration of Eq. (10) is affected by the number of iterations. As a result, Landweber's regularization in Eq. (10) converges to a useless solution owing to the amplification of hidden noise.

The problem now is how to select an appropriate number of iterations to obtain the optimal solution. In this study, an L-curve criterion [21] is used to obtain the appropriate number of iterations. The L-curve is a log-log plot of the norm of a regularized solution versus the norm of the corresponding residual because the number of iterations is varied. A log-log plot is represented as follows:

$$\left(\log \left\| Lu_m - \eta^{\delta} \right\|_2, \quad \log \left\| u_m \right\|_2 \right) \tag{18}$$

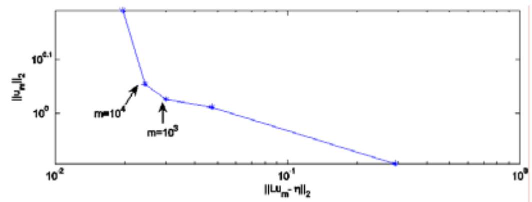


Fig. 5. Graphical illustration of the L-curve for $\gamma = -1$.

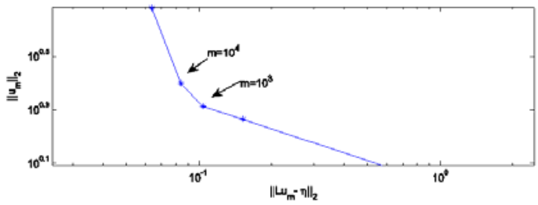


Fig. 6. Graphical illustration of the L-curve for $\gamma = -3$.

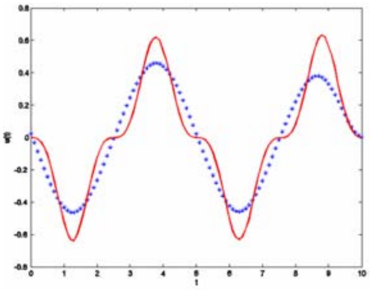
This curve exhibits a typical "L" shape, and the optimal value for the number of iterations is considered the one corresponding to the corner of the curve.

Figs. 5 and 6 show the L-curve criteria corresponding to the cases of $\gamma = -1$ and -3 when $\delta = 0.0096$: λ is taken as 0.1 for the iteration (10). The optimal numbers of iterations appear to be relatively clear at the corners of the L-curves as shown in Figs. 5 and 6: the optimal number of iterations $m = 10^4$ when $\gamma = -1$ and the optimal number of iterations $m = 10^4$ when $\gamma = -3$.

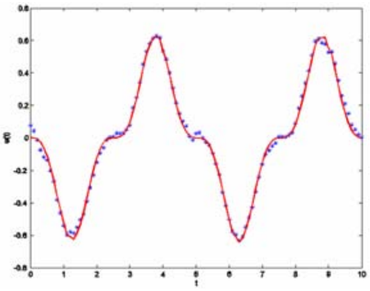
6.4 Polynomial approximation: Functional form of nonlinear restoring

The convergence behavior of solutions in iteration (10) is illustrated for different iteration numbers in Figs. 7 and 8: the solid lines denote the exact result of the graph for $u(t) = h[y(t)]$ in Eq. (6) where $h(y) = \gamma y^3$ in Eq. (14). It can be seen that small iterations yield a poor approximation. However, when the number of iterations exceeds a certain threshold, the solution worsens. We choose the value of the corner of L-curve as the optimal iteration number [21]. Nonlinear restoring forces $h(y)$ are recovered and depicted in Figs. 9 and 10, which are fairly accurate compared with the exact results of $h(y) = \gamma y^3$ in Eq. (14), regardless of the nonlinear parameters of $\gamma = -1$ and -3 in Duffing's Eq. (13).

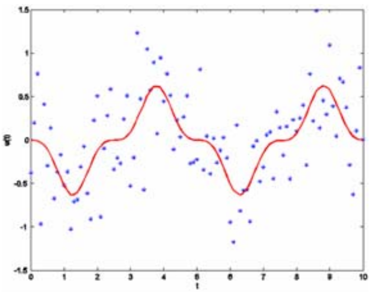
Sometimes it may be convenient to represent nonlinear restoring characteristics through polynomials rather than a set of data. For this purpose, we attempt to approximate the nonlinear restoring by a polynomial $p(y)$ of order n using the recovered



(a) $m = 10$

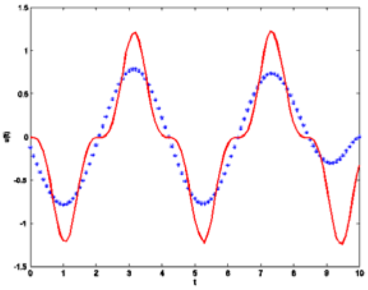


(b) $m = 10^4$

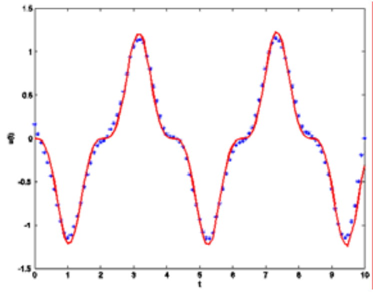


(c) $m = 10^6$

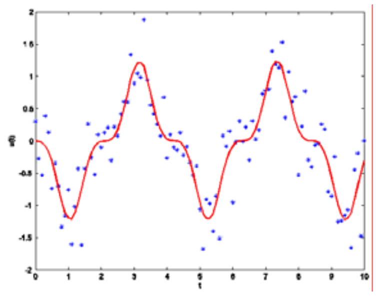
Fig. 7. Convergence behavior of $u_m(t)$ (dotted lines) in Eq. (10) for different iteration numbers m ($\gamma = -1$). The solid red lines indicate the exact solution of the graph for $u(t) = h[y(t)]$ in Eq. (6), where $h(y) = \gamma y^3$ in Eq. (14).



(a) $m = 10$



(b) $m = 10^4$



(c) $m = 10^6$

Fig. 8. Convergence behavior of $u_m(t)$ (dotted lines) in Eq. (10) for different iteration numbers m ($\gamma = -3$). The solid red lines indicate the exact solution of the graph for $u(t) = h[y(t)]$ in Eq. (6) where $h(y) = \gamma y^3$ in Eq. (14).

data of nonlinear restoring forces:

$$p(y) = a_0 + a_1 y + a_2 y^2 + ... + a_n y^n \quad (19)$$

In this study, the least squares method is applied to calculate the coefficient of Eq. (19). We minimize the residual in Eq. (20) to obtain the coefficients. The minimization is attained by differentiations of the residual.

$$\sum_{i=1}^{N} (h(y_i) - p(y_i))^2 \quad (20)$$

Figs. 11 and 12 show polynomial approximations of the third order for nonlinear restoring forces. Figs. 13 and 14 show the norm between the recovered nonlinear restoring forces $h(y)$ and the polynomial approximation $p_n(y)$ with respect to the order n of the polynomial. Figs. 13 and 14 show that the norm difference is dramatically decreased from the third order. Thus, it may be enough to take the third-order approximation, at least, for the present numerical experiment.

Finally, we resimulate the motion responses, displacement, and velocity by using the identified results of nonlinear restoring forces. A Runge-Kutta integration scheme is employed for the resimulation. Figs. 15 and 16 show that the resimulated results are fairly accurate compared with that of the exact solution.

For the explicit representation regarding the com-

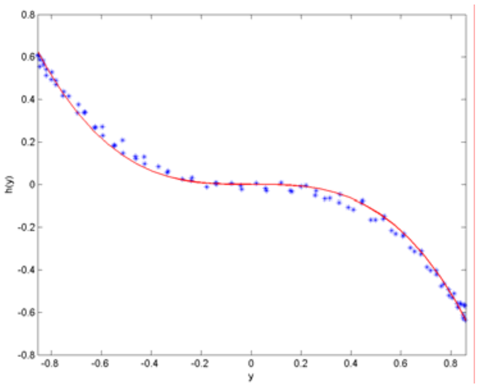


Fig. 9. Recovered nonlinear restoring force $h(y)$ for $\gamma = -1$. The solid red line denotes the exact nonlinear restoring force and the asterisks denote the recovered nonlinear restoring force.

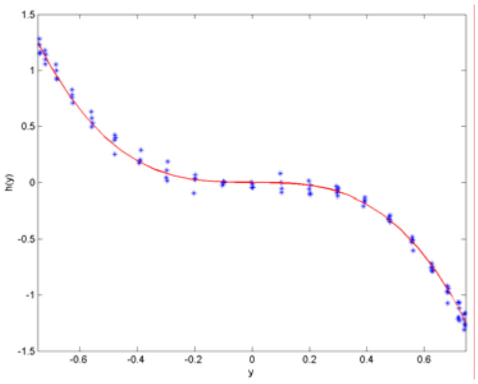


Fig. 10. Recovered nonlinear restoring force $h(y)$ for $\gamma = -3$. The solid red line denotes the exact nonlinear restoring force and the asterisks denote the recovered nonlinear restoring force.

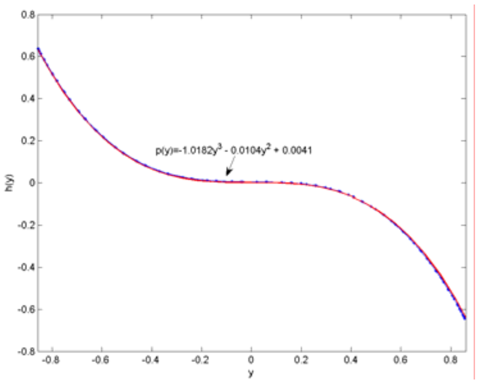


Fig. 11. Polynomial approximation of the identified functional form of the nonlinear restoring force for $\gamma = -1$. The solid red line denotes the exact nonlinear restoring force and the dotted line denotes the recovered nonlinear restoring force.

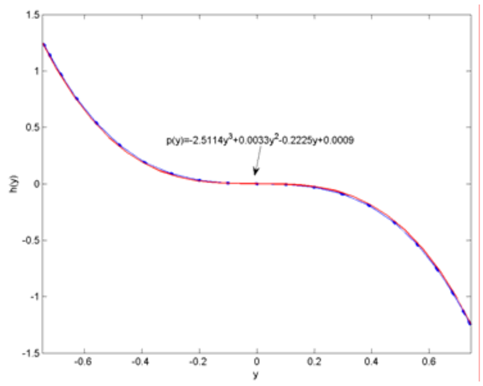


Fig. 12. Polynomial approximation of the identified functional form of the nonlinear restoring force for $\gamma = -3$. The solid red line denotes the exact nonlinear restoring force and the dotted line denotes the recovered nonlinear restoring force.

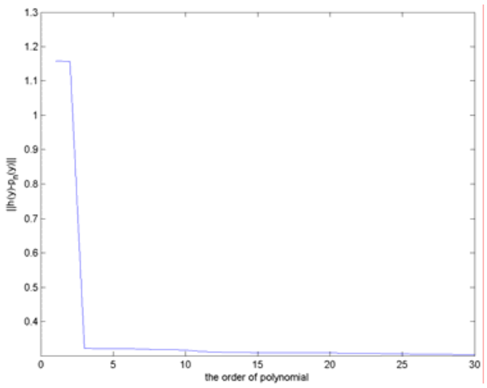


Fig. 13. Norm between the recovered nonlinear restoring forces $h(y)$ and the polynomial approximation $P_n(y)$ with respect to the order of polynomial $\gamma = -1$.

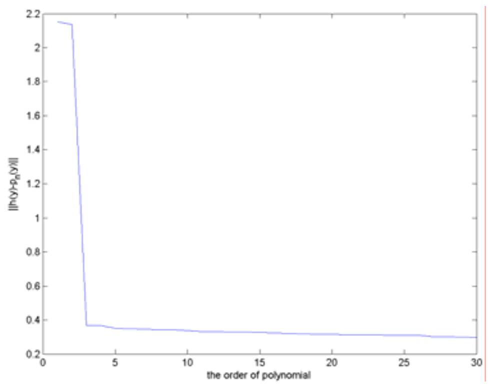


Fig. 14. Norm between the recovered nonlinear restoring forces $h(y)$ and the polynomial approximation $P_n(y)$ with respect to the order of polynomial $\gamma = -3$.

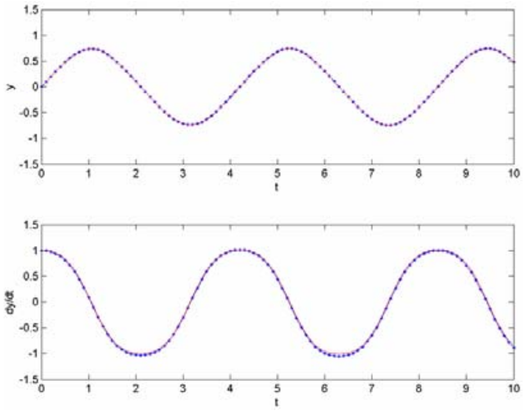


Fig. 15. Resimulated motion responses, displacement, and velocity using the identified results of nonlinear restoring forces in Fig. 11 for $\gamma = -1$. The solid red line denotes the exact nonlinear restoring force and the dotted line denotes the resimulated motion responses.

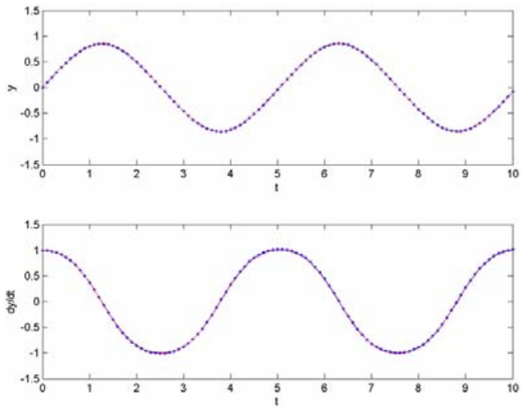


Fig. 16. Resimulated motion responses, displacement, and velocity using the identified results of nonlinear restoring forces in Fig. 12 for $\gamma = -3$. The solid red line denotes the exact nonlinear restoring force and the dotted line denotes the resimulated motion responses.

parison of the accuracy of the proposed method, we conduct several numerical experiments with different noise levels. The corresponding results are summarized in Table 1, in which δ denotes the noise level and,

$\|h_{num} - h_{exact}\|_2$: L_2 norm difference between identified nonlinear and exact nonlinear restoring forces,

$\|y_{resim} - y_{exact}\|_2$: L_2 norm difference between resimulated and exact displacements,

$\|\dot{y}_{resim} - \dot{y}_{exact}\|_2$: L_2 norm difference between resimulated and exact velocities.

Table 1. Comparison of the level of accuracy.

	δ	$\|h_{num} - h_{exact}\|_2$	$\|y_{resim} - y_{exact}\|_2$	$\|\dot{y}_{resim} - \dot{y}_{exact}\|_2$
$\gamma = -1$	0.0096	0.0118	0.0027	0.072
	0.0973	0.4512	0.1633	0.2400
	0.02794	1.1023	0.1915	0.4122
$\gamma = -3$	0.0099	0.1109	0.0815	0.1395
	0.0992	0.3483	0.1925	0.2844
	0.3257	0.9308	0.2205	0.4869

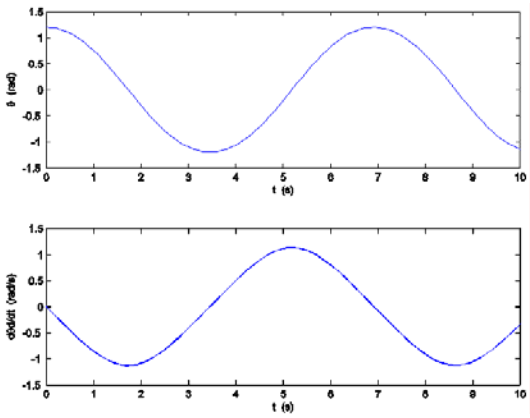


Fig. 17. Motion responses of the simple nonlinear pendulum in Eq. (21) with initial conditions in Eq. (22).

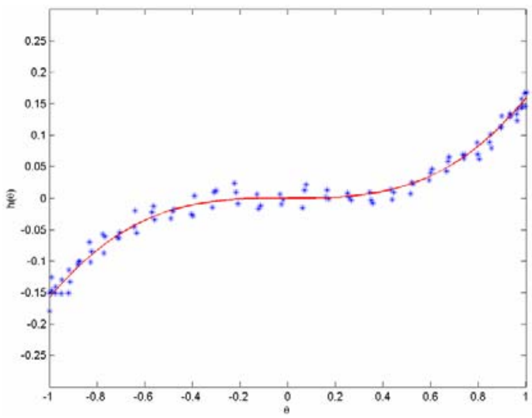


Fig. 18. Recovered nonlinear restoring $h(\theta)$. The solid red line denotes the exact nonlinear restoring force and the asterisks denote the recovered nonlinear restoring force.

6.5 Other nonlinear equation: Nonlinear pendulum

Finally, we examine an additional example, which has the other nonlinearity of restoring force:

$$\ddot{\theta} + \kappa \sin\theta = 0 \tag{21}$$

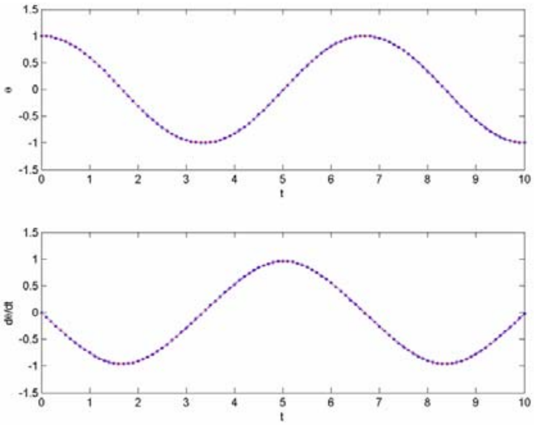


Fig. 19. Resimulated motion responses, displacement, and velocity using the identified results of nonlinear restoring forces in Fig. 18. The solid red line denotes the exact nonlinear restoring force and the dotted line denotes the resimulated motion responses.

Eq. (21) governs a nonlinear large motion of a simple pendulum, in which θ and κ correspond to (angular) displacement and constant, respectively. From (1) and (21), it is clear that $h(\theta) = \kappa\theta - \kappa \sin\theta$. For convenience, we impose the initial conditions:

$$\theta(0) = 1, \dot{\theta}(0) = 0 \tag{22}$$

and the coefficient κ is normalized as a unit. Figs. 17-19 illustrate the numerical results concerning the nonlinear pendulum. The proposed method also gives quite accurate results as confirmed by the numerical results in Figs. 17-19.

7. Conclusion

In this paper, an inverse problem is studied to identify the functional form of nonlinear restoring forces in nonlinear oscillatory motion. The problem is mathematically involved in solving the first-kind integral equation, resulting in a numerical instability that will influence the performance of the present identification. The identification is an ill-posed inverse problem in the sense that its solution lacks stability properties. This implies that a small amount of noisy data can be considerably amplified and may lead to unreliable solutions. To deal with measured data deteriorated by noise, Landweber's regularization method is introduced to overcome the numerical instability. An L-curve criterion, combined with the regularization, is applied to the identification process to provide an optimal choice of the number of iterations for the regularization. The workability of the proposed identification scheme is illustrated with the example of the Duffing's equation. It is shown that this identification can be successfully used to determine the functional form of the nonlinear restoring forces in a stable and accurate manner, regardless of γ = -1 and -3. To prove the usefulness of the presented method, the other kind of nonlinear equation, the simple nonlinear pendulum, is also examined.

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